HEAT TRANSFER DURING FILTRATION IN A HETEROGENEOUS MEDIUM WITH INPUT CONDITIONS AT A MOVING BOUNDARY

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Theoretical results on the heat transfer in combustion-product filtration in the rock-collapse zone behind a borehole-damaged coal bed are outlined. A sufficiently simple engineering solution of the problem is obtained by means of an equivalent heat-conduction equation, and the limiting temperatures corresponding to different parameter values are analyzed.

The thermodynamic efficiency with which the chemical energy of coal burnup is used in traditional methods of coal–gas production is low. The energy losses in the depths of the earth associated with heating of the coal bed running through the rock, evaporation of the water which is present, and leakage of the gas which forms amount to around 30%, and result in a low-efficiency of underground gas-generating stations: 50% on average. Recovery of the heat losses could significantly increase the efficiency of thermal utilization of the coal.

Beginning in the 1970s, borehole processing of coal at deep levels has been developed at the Leningrad Mining Institute, in combination with the recovery of geothermal energy [1].

With some modifications, retaining the basic idea of coal combustion in underground conditions (supplying water where necessary), with subsequent or simultaneous recuperation of the physical heat stored in the collapsed area, this technology may be used for beds at any depth.

In the complex system for the recovery of coal and geothermal reserves, certain characteristic zones may be distinguished in the set of complex structures of heat-transfer, chemical-conversion, and phase-transformation surfaces: the zone where there is practically no chemical reaction; the combustion zone; the zone with heat transfer by the vapor–gas mixture in the gas-production channel, and the zone with collapsed rock. The basic losses of physical heat occur in the zone of collapse and stratification of the rock layer. The reaction products or, with water supply to the combustion zone, the vapor–gas mixture moves along the gas-production channel and then through the collapsed-rock zone. The heat losses in the channel were estimated in [2]. The aim of the present work is to determine the gas temeprature on leaving the collapse zone, permitting the estimation of the heat losses in this zone; subsequent recovery of this heat significantly influences the economic and energy indices of system operation.

For the heat-transfer zone, periodic collapse of the roof of the bed following the passage of the combustion front and the energy carrier is characteristic. The size of the blocks is sufficiently large, and therefore heat transfer occurs in a significantly heterogeneous medium. The description of heat transfer in permeable media was considered in [3, 4]. In contrast to traditional formulations and solutions to conjugate heat-transfer problems in filtration [4-8], the input conditions in the given problem are specified at a moving boundary; there is constant increase in volume of the heterogeneous volume and hence in the heat-transfer surface.

The situation in which there is nonsteady heat transfer in a heterogeneous medium with motion of one of the boundaries of the filtration region is also realized in various processes in the metallurgical and food industry, chemical technology, geotechnology, etc. Therefore, the solution of the problem in a formulation with moving boundary conditions is of interest for many applications.

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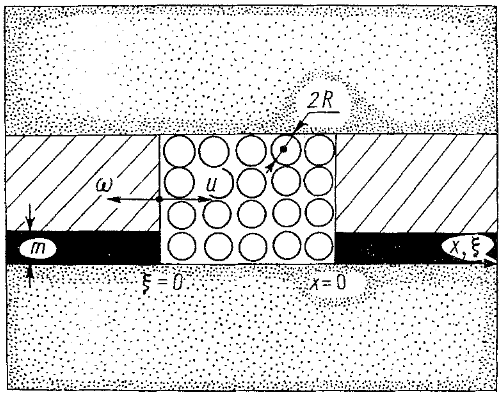

Fig. 1. Model of heat-transfer problem in heterogeneous medium.

The gas motion is regarded as one-dimensional filtration in a heterogeneous medium. Since the time of bed treatment considerably exceeds the periods of collapse, it is assumed that the velocity at which the zone boundary moves is specified and equal to the velocity at which the combustion face moves. The heterogeneous medium may be modeled by a system of parallel cracks or a layer of spherical elements with equivalent radius R. Boundary conditions of type I or III (depending on the ratio of Bi and Nu) are specified at the interface between the media, and the symmetry condition is imposed at the center of the layer (bed) particles. Since the collapse zone is sufficiently large and the surface area of elements of this zone is several times greater than the heat-transfer surface with the intact rock mass, heat losses in the surrounding medium are disregarded. A possible model of the heterogeneous medium and scheme of gas motion is shown in Fig. 1. The system of equations describing heat transfer in such conditions includes: an equation for the gas phase with an internal heat source, the differential heat-conduction equation for solids of the simplest form, and the uniqueness conditions

$$\frac{\partial t(x,\ \tau)}{\partial \tau} + u\frac{\partial t(x,\ \tau)}{\partial x} = \frac{\sigma q}{c_g \rho_g}; \tag{1}$$

$$\frac{\partial T(x,\ y,\ \tau)}{\partial \tau} = a_r\left(\frac{\partial^2 T(x,\ y,\ \tau)}{\partial y^2} + \frac{\Gamma}{y}\frac{\partial T(x,\ y,\ \tau)}{\partial y}\right); \tag{2}$$

$$q = \lambda_r \frac{\partial T(x,\ y,\ \tau)}{\partial y}\bigg|_{y=R}; \tag{3}$$

$$x + \omega\tau \geqslant (\omega + u)\tau,\ t = T = T_{in}; \tag{4}$$

$$x = -\omega\tau,\ \tau > 0,\ t = t_0,\ T = T_{in}; \tag{5}$$

$$y = 0,\ \tau > 0,\ \frac{\partial T}{\partial y} = 0,\ T \neq \infty. \tag{6}$$

At the interface, boundary conditions of the first kind

$$\tau > 0,\ x > -\omega\tau,\ T(x,\ R,\ \tau) = t(x,\ \tau) \tag{7}$$

or the third kind

$$\tau > 0,\ x > -\omega\tau,\ \lambda_r \frac{\partial T(x,\ y,\ \tau)}{\partial y}\bigg|_{y=R} = \alpha\,[t(x,\ \tau) - T(x,\ y,\ \tau)] \tag{8}$$

are specified. Here Γ is a constant: $\Gamma = 0 (y \equiv y)$ for a plate and $\Gamma = 2 (y \equiv r)$ for a sphere.

Converting to a coordinate system associated with the moving boundary, where $\xi = x + \omega\tau$, $\tau^* = \tau - \xi/(u + \omega)$, Eqs. (1)-(8) may be rewritten in the dimensionless form

$$\frac{\partial \theta}{\partial X} = G\frac{\partial \vartheta}{\partial Y}\bigg|_{Y=1}; \tag{9}$$

$$\frac{\partial \vartheta}{\partial\,\mathrm{Fo}^*} = \frac{\partial^2 \vartheta}{\partial Y^2} + \frac{\Gamma}{Y}\frac{\partial \vartheta}{\partial Y}; \tag{10}$$

$$\mathrm{Fo}^* \leqslant 0,\ X > 0,\ \theta = \vartheta = 0; \tag{11}$$

$$X = 0,\ \mathrm{Fo}^* > 0,\ \theta = 1,\ \vartheta = 0; \tag{12}$$

$$Y = 0,\ \mathrm{Fo}^* > 0,\ \frac{\partial \vartheta}{\partial Y} = 0,\ \vartheta \neq \infty; \tag{13}$$

$$Y = 1,\ X > 0,\ \mathrm{Fo}^* > 0 \begin{cases} \vartheta = \theta \ \text{(condition of the first kind)}, \\ \left.\dfrac{\partial \vartheta}{\partial Y}\right|_{Y=1} = \mathrm{Bi}\,(\theta - \vartheta)|_{Y=1} \ \text{(condition of the third kind)}. \end{cases} \tag{14}$$

To find the heat flux in Eq. (9), Eq. (10) must be solved for the corresponding form of the elements of the heterogeneous medium.

For boundary conditions of the first kind

$$\left.\frac{\partial \vartheta}{\partial Y}\right|_{Y=1} = 2\sum_{n=1}^{\infty} \frac{\partial}{\partial\,\mathrm{Fo}^*} \int_0^{\mathrm{Fo}^*} \theta\,(\mathrm{Fo}_*^*) \exp\,[-\mu_n^2\,(\mathrm{Fo}^* - \mathrm{Fo}_*^*)]\, d\,\mathrm{Fo}_*^*, \tag{15}$$

where μ_n is a root of the characteristic equation $W_\Gamma(\mu_n) = 0$.

For boundary conditions of the third kind

$$\left.\frac{\partial \vartheta}{\partial Y}\right|_{Y=1} = \sum_{n=1}^{\infty} \frac{2\,(-1)^{n+1}\,\mathrm{Bi}^2}{[\mu_n^2 + \mathrm{Bi}^2 + (1-\Gamma)\,\mathrm{Bi}]} \frac{\partial}{\partial\,\mathrm{Fo}^*} \int_0^{\mathrm{Fo}^*} \theta\,(\mathrm{Fo}_*^*) \exp\,[-\mu_n^2\,(\mathrm{Fo}^* - \mathrm{Fo}_*^*)]\, d\,\mathrm{Fo}_*^*, \tag{16}$$

where μ_n is the root of the characteristic equation $W_\Gamma(\mu_n)V_\Gamma(\mu_n) = \mu_n/\mathrm{Bi}$.

For plates

$$\Gamma = 0,\ W_0(\mu) = \cos\mu;\ V_0(\mu) = \sin\mu;$$

and for spheres

$$\Gamma = 2,\ W_2(\mu) = \frac{\sin\mu}{\mu};\ V_2(\mu) = \frac{\sin\mu - \mu\cos\mu}{\mu^2}.$$

Taking account of these expressions, Eq. (9) with the corresponding conditions in Eqs. (11)-(13) takes the form

$$\frac{\partial \theta}{\partial X} = G\sum_{n=1}^{\infty} P_n \frac{\partial}{\partial\,\mathrm{Fo}^*} \int_0^{\mathrm{Fo}^*} \theta\,(\mathrm{Fo}_*^*) \exp\,[-\mu_n^2\,(\mathrm{Fo}^* - \mathrm{Fo}_*^*)]\, d\,\mathrm{Fo}_*^*, \tag{17}$$

where $P_n = 2$ for boundary conditions of the first kind and

$$P_n = \frac{2\,\mathrm{Bi}}{\mu_n^2 + \mathrm{Bi} + (1-\Gamma)\,\mathrm{Bi}}$$

for boundary conditions of the third kind.

In [7, 9], a method of solving heat-transfer equations with filtration was proposed and tested. The basis of this method, the equivalent-equation method, is that the mutual heat transfer between the liquid and the structural elements of the medium may be described by a differential approximation of the Duhamel integral on the right-hand side of Eq. (17).

The use of the method in heat and mass-transfer theory, as well as more general problems of reducing such systems to an equation with one dependent variable, the validity conditions of the approximation, and the character of the resulting errors, has been investigated by Buevich et al., for example, in [3, 10-12].

This method yields an equation with initial and boundary conditions in which it is taken into account that the heat content of the filtration zone is finite and there is no temperature variation of the phases at sufficiently large times

$$\frac{\partial \theta}{\partial\,\mathrm{Fo}^*} + \frac{1}{AG}\frac{\partial \theta}{\partial X} = \frac{B}{A}\frac{\partial^2 \theta}{\partial\,\mathrm{Fo}^{*2}}; \tag{18}$$

$$\mathrm{Fo}^* \leqslant 0,\ X > 0,\ \theta = \vartheta = 0; \tag{19}$$

$$X = 0,\ \mathrm{Fo}^* > 0,\ \theta = 1,\ \vartheta = 0; \tag{20}$$

TABLE 1. Coefficients A and B for Calculating the Filtrational Heat Transfer with Boundary Conditions of the Third Kind

Filtration in parallel-crack system			Filtration in layer of spheres		
Bi	A	B	Bi	A	B
0,001	1,000	964,81	0,005	0,339	22,770
0,01	1,000	102,89	0,02	0,336	6,651
0,08	1,000	12,85	0,08	0,334	1,411
0,5	1,000	2,336	0,5	0,334	0,245
1,0	1,000	1,335	1,0	0,333	0,133
10	0,997	0,433	10	0,331	0,033
40	0,984	0,359	50	0,318	0,025
100	0,971	0,343	100	0,313	0,024

$$\mathrm{Fo}^* \to \infty,\ X > 0,\ \theta \quad 1. \tag{21}$$

The solution of Eqs. (18)-(21) for the temperature distribution takes the form

$$\theta(X,\ \mathrm{Fo}^*) = 1 - \frac{1}{2}\left\{\mathrm{erfc}\left[\frac{\mathrm{Fo}^* - AGX}{2\sqrt{BGX}}\right] + \right.$$

$$\left. + \exp\left[A\,\mathrm{Fo}^*/B\right]\mathrm{erfc}\left[\frac{\mathrm{Fo}^* + AGX}{2\sqrt{BGX}}\right]\right\},\quad A = \sum_{n=1}^{\infty}\frac{P_n}{\mu_n^2};\ B = \sum_{n=1}^{\infty}\frac{P_n}{\mu_n^4}. \tag{22}$$

Values of A and B for various conditions are given in Table 1.

Letting x = 0 and substituting

$$\mathrm{Fo}^* = \mathrm{Fo}\left(\frac{u}{u+\omega}\right) \text{ and } X = \mathrm{Fo}\left(\frac{\omega}{u+\omega}\right),$$

into Eq. (22), the gas temperature at the exit from the collapse zone is found:

$$\theta(0,\ \mathrm{Fo}^*) = \theta_{\mathrm{out}} = 1 - \frac{1}{2}\left\{\mathrm{erfc}\left[\frac{1}{2}\left(\sqrt{k} - \frac{1}{\sqrt{k}}\right)\sqrt{\frac{A}{B}\mathrm{Fo}^*}\right] + \right.$$

$$\left. + \exp\left(\frac{A}{B}\mathrm{Fo}^*\right)\mathrm{erfc}\left[\frac{1}{2}\left(\sqrt{k} + \frac{1}{\sqrt{k}}\right)\sqrt{\frac{A}{B}\mathrm{Fo}^*}\right]\right\}. \tag{23}$$

Here k = u/(ωAG).

The local gas velocity u in the collapse zone is due to the mass of gas formed as a result of reaction and the geometric parameters of the filtrational collector and consequently

$$\chi\rho_c m\omega = u m_{\phi}\varepsilon\rho_r. \tag{24}$$

Hence

$$k = \frac{\chi\rho_c c_g n}{3A\rho_r c_r m_d\,(1-\varepsilon)}. \tag{25}$$

Undoubtedly, the gas formed is not distributed over the whole region of collapse; therefore, m_d is taken to be the height of the collapse-zone model in which motion in the given technological conditions is taken into account using the experimental capture coefficient. The parameter k characterizes the ratio of growth rates of the volume specific heats of the two phases of the heterogeneous medium and the heat-transfer conditions at their interface.

Analysis of Eq. (23) permits the estimation of the asymptotic gas temperature on leaving the collapse zone as $\mathrm{Fo}^* \to \infty$:

$$\min\left(\sqrt{k} + \frac{1}{\sqrt{k}}\right) = 2, \tag{26}$$

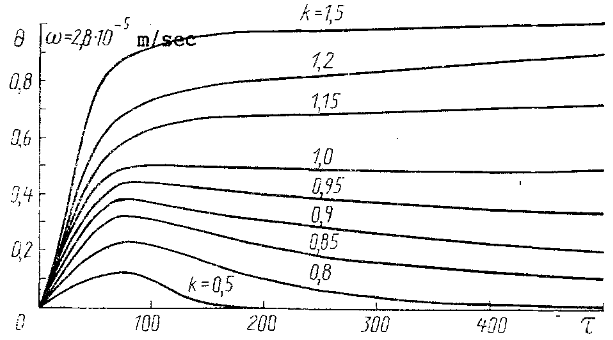

Fig. 2. Variation in gas temperature on leaving collapse zone; boundary conditions of the first kind: A = 0.302; B = 0.022. τ, h.

$$\text{when } k<1,\ \left(\sqrt{k}-\frac{1}{\sqrt{k}}\right)<0\ \ \theta_{out}\to 0;$$

$$\text{when } k=1,\ \left(\sqrt{k}-\frac{1}{\sqrt{k}}\right)=0\ \ \theta_{out}=0{,}5;$$

$$\text{when } k>1,\ \left(\sqrt{k}-\frac{1}{\sqrt{k}}\right)>0\ \ \theta_{out}\to 1.$$

Using Eq. (23), it is sufficiently simple to determine the space–time distribution of the energy–carrier temperature in the system and to obtain the solution for the solid-phase temperature [4]. In addition, knowing the combustion temperature and the gas temperature at the collector exit (Fig. 2), as well as the flow rate of combustion products (or the vapor–gas mixture), the heat losses in this zone may be estimated and, on this basis, the mean temperature of the rock may be calculated. This is the initial parameter for calculating the temperature field of the heat carrier in the next technological stage: the creation of a geothermal circulation system in the collapse zone above the treated coal bed.

NOTATION

x, y, r, longitudinal, transverse, and radial coordinates; τ, time; T, gas-phase temperature; T_{in}, initial temperature; t_0, gas temperature at input; u, real gas velocity; ω, velocity of motion of input-condition boundary; a_r, λ_r, c_r, ρ_r, thermal diffusivity, thermal conductivity, specific heat, and density of solid phase (rock); ρ_g, c_g, density and specific heat of gas; ρ_c, density of coal; χ, stoichiometric coefficient; ε, porosity of layer; σ, surface area of solid phase per unit volume of gas; m_d, width of collapse zone; m, width of coal bed; α, heat-transfer coefficient; $\theta = (t - T_{in})/(t_0 - T_{in})$, dimensionless gas temperature; $\vartheta = (T - T_{in})/(t_0 - T_{in})$, dimensionless solid-phase temperature. Dimensionless complexes: $Y = y/R$; $X = \alpha_r\xi/R^2(u + \omega)$; $G = R\sigma\rho_r c_r/\rho_g c_g$; $\sigma = (\Gamma + 1)(1 - \varepsilon)/R\varepsilon$; $Fo = a_r\tau/R^2$; $Fo^* = Fo - X$; $Bi = \alpha R/\lambda_r$; $Nu = \alpha R/\lambda_g$.

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APPROXIMATE APPROACH TO SOLUTION OF THREE-DIMENSIONAL HEAT CONDUCTION PROBLEMS

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An approximate approach is presented for the solution of a three-dimensional heat conduction problem for one type of heat-protective coating frequently encountered in practice. The method proposed, based on the reduction of the initial three-dimensional problem to a set of two problems of less dimensionality, makes it possible, with satisfactory precision, to shorten substantially the computer time spent in determining thermal conditions of elements of a given design.

Aircraft bodies are, at the present time, in the majority of cases, thin-walled supporting metallic shells coated on the outside and (or) inside with a many-layered heat insulation. Large flight speeds at nonzero angles of attack give rise to high thermal flow densities and a resulting nonuniformity in the distribution of these flows over the structural surface. Flow-on and flow-off directions, and different nodes of pressure and rarefaction at locations shaded by aerodynamic elements, give rise to zones of minimum and peak loads on an object. To correctly estimate the thermal state of an aircraft, it is necessary under these conditions to consider the heat conduction equation in three dimensions. A rigorous solution of a similar problem by traditional numerical methods becomes in some cases (in carrying-out algorithms in a "real time scale" or in handling large-scale computational systems) impractical due to the resulting high expenditure of machine time. In the present paper we present one of the possible approaches to solving a nonstationary three-dimensional heat conduction problem, an approach which makes it possible, with satisfactory precision, to obtain results with substantially less computational time spent in determining thermal conditions of elements of a structural object.

A basic feature of the engineering method presented here is the fact that three-dimensional calculation of heat conduction of the shell is reduced to a set of two problems: two-dimensional initial heating of the supporting framework, with neglect of temperature drop over its thickness, and a one-dimensional initial heating of the thermal insulation in a direction perpendicular to the surface of the metallic layer. Here a direct calculation of the temperature field over the structure is carried out while solving the indicated one-dimensional problem using values of a source function in place of the metallic layer arrangement. The source function is calculated within the scope of the two-dimensional heat conduction problem and is connected with thermal overcurrents in longitudinal and transverse directions. In particular, the source function for the case involving calculation of an element of an axially-symmetric shell has the following form:

$$Q^S = \delta\left(\frac{Q_{i-1} - Q_i}{l} - \frac{Q_{i-1} + Q_i}{2r}\,\mathrm{tg}\,\theta + \frac{Q_{j-1} - Q_j}{r d\varphi}\right).$$

The physical justification for use of such an approximate approach is based on a qualitative difference in the coefficients of thermal conductivity of elements of the structure of the type considered and also on the relatively small thicknesses of the metallic layer.

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